... (approx.).

Example 8·6. *A department in a works has 10 machines which may need adjustment from time to time during the day. Three of these machines are old, each having a probability of 1/11 of needing adjustment during the day, and 7 are new, having corresponding probabilities of 1/21.*

Assuming that no machine needs adjustment twice on the same day, determine the probabilities that on a particular day

(i) just 2 old and no new machines need adjustment.

(ii) If just 2 machines need adjustment, they are of the same type.

Solution. Let p_1 = Probability that an old machine needs adjustment $= \frac{1}{11} \Rightarrow q_1 = \frac{10}{11}$.

and p_2 = Probability that a new machine needs adjustment $= \frac{1}{21} \Rightarrow q_2 = \frac{20}{21}$.

Then $P_1(x)$ = Probability that 'x' old machines need adjustment

$$= \binom{3}{x} p_1^x q_1^{3-x} = \binom{3}{x} \left(\frac{1}{11}\right)^x \left(\frac{10}{11}\right)^{3-x} ; x = 0, 1, 2, 3$$

and $P_2(x)$ = Probability that 'x' new machines need adjustment

$$= \binom{7}{x} p_2^x q_2^{7-x} = \binom{7}{x} \left(\frac{1}{21}\right)^x \left(\frac{20}{21}\right)^{7-x} ; x = 0, 1, 2, \ldots, 7$$

(i) The probability that just two old machines and no new machine need adjustment is given (by the compound probability theorem) by the expression :

$$P_1(2) \cdot P_2(0) = \binom{3}{2} \left(\frac{1}{11}\right)^2 \left(\frac{10}{11}\right) \left(\frac{20}{21}\right)^7 = 0{\cdot}016 \qquad \ldots (1)$$

(ii) Similarly, the probability that just 2 new machines and no old machine need adjustment is :

$$P_1(0).\ P_2(2) = \left(\frac{10}{11}\right)^3 \times \binom{7}{2} \left(\frac{1}{21}\right)^2 \left(\frac{20}{21}\right)^5 = 0{\cdot}028 \qquad \ldots (2)$$

The probability that 'if just two machines need adjustment, they are of the same type' is the same as the probability that 'either just 2 old and no new or just 2 new and no old machines need adjustment'.

	$\binom{12}{6} = 924$	Total	4,096

Example 8·22. *Seven coins are tossed and number of heads noted. The experiment is repeated 128 times and the following distribution is obtained :*

No. of heads	0	1	2	3	4	5	6	7	*Total*
Frequencies	7	6	19	35	30	23	7	1	128

Scanned with CamScanner

Fit a binomial distribution assuming, (i) The coin is unbaised. (ii) The nature of the coin is not known. (iii) Probability of a head for four coins is 0·5 and for the remaining three coins is 0·45.

Solution. In fitting binomial distribution, first of all the mean and variance of the data are equated to np and npq respectively. Then the expected frequencies are calculated from these values of n and p. Here $n = 7$ and $N = 128$.

Case I. *When the coin is unbiased :* $\quad p = q = \frac{1}{2}, \qquad \frac{p}{q} = 1$

$$p(0) = q^n = \left(\frac{1}{2}\right)^7 = \frac{1}{128} \quad \text{so that} \quad f(0) = Nq^n = 128\left(\frac{1}{2}\right)^7 = 1$$

Using the recurrence formula (8·15), the various probabilities, *viz.*, $p(1), p(2), \ldots$ can be easily calculated as shown below :

COMPUTATION OF EXPECTED BINOMIAL FREQUENCIES

x	f	fx	$\frac{n-x}{x+1}$	$\frac{n-x}{x+1} \cdot \frac{p}{q}$	*Expected frequency* $f(x) = Np(x)$
0	7	0	7	7	$f(0) = Np(0) = 1$
1	6	6	3	3	$f(1) = 1 \times 7 = 7$
2	19	38	$\frac{5}{3}$	$\frac{5}{3}$	$f(2) = 7 \times 3 = 21$
3	35	105	1	1	$f(3) = 21 \times \frac{5}{3} = 35$
4	30	120	$\frac{3}{5}$	$\frac{3}{5}$	$f(4) = 35 \times 1 = 35$
5	23	115	$\frac{1}{3}$	$\frac{1}{3}$	$f(5) = 35 \times \frac{3}{5} = 21$
6	7	42	$\frac{1}{7}$	$\frac{1}{7}$	$f(6) = 21 \times \frac{1}{3} = 7$
7	1	7			$f(7) = 7 \times \frac{1}{7} = 1$
Total	128	433			

Case II. *When the nature of the coin is not known*, then

$$\text{Mean} = np = \bar{x} = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{433}{128} = 3{\cdot}3828\,; \qquad n = 7$$

$$\therefore \quad p = \frac{3{\cdot}3828}{7} = 0{\cdot}48326 \quad \text{and} \quad q = 1 - p = 0{\cdot}51674, \quad \Rightarrow \quad \frac{p}{q} = 0{\cdot}93521$$

$$f(0) = Nq^7 = 128\,(0{\cdot}5167)^7 = 1{\cdot}2593 \text{ (using logarithms)}$$

COMPUTATION OF EXPECTED BINOMIAL FREQUENCIES

x	$\frac{n-x}{x+1}$	$\frac{n-x}{x+1} \cdot \frac{p}{q}$	*Expected frequency* $f(x) = N\,p(x)$
0	7	6·54647	$f(0) = Np(0) = 1{\cdot}2593 \simeq 1$
1	3	2·80563	$f(1) = 1{\cdot}2593 \times 6{\cdot}54647 = 8{\cdot}2438 \simeq 8$
2	$\frac{5}{3}$	1·55868	$f(2) = 2{\cdot}80563 \times 8{\cdot}2438 = 23{\cdot}129 \simeq 23$
3	1	0·93521	$f(3) = 1{\cdot}55868 \times 23{\cdot}129 = 36{\cdot}05 \simeq 34$
4	$\frac{3}{5}$	0·56113	$f(4) = 0{\cdot}93521 \times 36{\cdot}05 = 33{\cdot}715 \simeq 34$
5	$\frac{1}{3}$	0·31174	$f(5) = 0{\cdot}56113 \times 33{\cdot}715 = 18{\cdot}918 \simeq 19$
6	$\frac{1}{7}$	0·13360	$f(6) = 0{\cdot}31174 \times 18{\cdot}918 = 5{\cdot}897 \simeq 6$
7			$f(7) = 0{\cdot}13360 \times 5{\cdot}897 = 0{\cdot}788 \simeq 1$

where n is the total frequency, i.e. the total number of sets of m trials each.

Example 8.1 Twelve dice were thrown 2,630 times and each time the number of dice which had 5 or 6 on the uppermost face was recorded. The results are shown in the following table :

Number of dice with 5 or 6 uppermost	0	1	2	3	4	5	6	7	8	9	10	11	12
Frequency	18	115	326	548	611	519	307	133	40	11	2	0	0

Graduate the observed distribution (a) with a binomial distribution for which p is unknown and (b) with a binomial distribution for which $p = \frac{1}{3}$.

Case 1 : Here to fit a binomial distribution, p has to be estimated from the observed distribution. The mean of the latter distribution is

$$\bar{x} = \frac{\sum_x xf_x}{n} = \frac{10{,}662}{2{,}630} = 4 \cdot 05399;$$

so the estimate of p is

$$\hat{p} = \frac{4 \cdot 05399}{12} = 0 \cdot 33783.$$

The probabilities $f(x)$ are calculated by using the relation

$$f(x) = \left(\frac{m - x + 1}{x} \times \frac{\hat{p}}{\hat{q}} \right) \times f(x-1),$$

for $x = 1, 2, \ldots\ldots, m$. Here

$$f(0) = \hat{q}^m,$$

or

$$\log f(0) = m \log \hat{q} = 12 \times \log 0 \cdot 66217$$

$$= \bar{3} \cdot 8516340 = \log 0 \cdot 0071061,$$

so that

$$f(0) = 0 \cdot 0071061.$$

Also, $\hat{p}/\hat{q} = 0 \cdot 51019.$

The subsequent calculations are shown in Table 8.2 :

A comparison of the last two columns of the table indicates that the fit has been quite satisfactory.

TABLE 8.2

FITTING A BINOMIAL DISTRIBUTION TO THE FREQUENCY DISTRIBUTION OF NUMBER OF DICE SHOWING 5 OR 6 IN 2,630 THROWS OF 12 DICE (p ESTIMATED FROM DATA)

x (1)	$\frac{m-x+1}{x}$ (2)	col. (2) × $\hat{p}/\hat{q}$ (3)	$f(x) = f(x-1)$ × col. (3) (4)	Expected frequency = n × col. (4) (5)	Observed frequency (6)
0	—	—	0·0071061	18·69	18
1	12	6·12228	0·0435055	114·42	115
2	5·5	2·80604	0·1220782	321·07	326
3	3·33333	1·70063	0·2076098	546·01	548
4	2·25	1·14793	0·2383215	626·79	611
5	1·6	0·81630	0·1945418	511·64	519
6	1·16667	0·59522	0·1157952	304·54	307
7	0·85714	0·43730	0·0506372	133·18	133
8	0·625	0·31887	0·0161467	42·47	40
9	0·44444	0·22675	0·0036613	9·63	11
10	0·3	0·15306	0·0005604	1·47	2
11, 12	—	—	0·0000363*	0·09	0
Total	—	—	1·0000000	2,630·00	2,630

*Obtained from the identity : $f(11) + f(12) = 1 - \sum_{x=0}^{10} f(x)$

Case 2 : Here the procedure is the same as in Case 1, but for p we now use its given value, 1/3. So

$$f(0) = q^m = (2/3)^{12}.$$

or

$$\log f(0) = 12(\log 2 - \log 3)$$

$$= \bar{3}\cdot 8869044 = \log 0\cdot 0077073,$$

giving

$$f(0) = 0\cdot 0077073.$$

We also note that

$$p/q = \tfrac{1}{2}.$$

The expected frequencies may then be calculated as in Case 1.

Here also a comparison of cols. (5) and (6) of the Table 8.3 shows that the fit has been fairly satisfactory, although it is less good than in Case 1.

TABLE 8.3

FITTING A BINOMIAL DISTRIBUTION TO THE FREQUENCY DISTRIBUTION OF NUMBER OF DICE SHOWING 5 OR 6 IN 2,630 THROWS OF 12 DICE ($p = \frac{1}{3}$)

x (1)	$\frac{m-x+1}{x}$ (2)	col. (2) × p/q (3)	$f(x) = f(x-1)$ × col. (3) (4)	Expected frequency $= n \times$ col. (4) (5)	Observed frequency (6)
0	—	—	0·0077073	20·27	18
1	12	6·00000	0·0462438	121·62	115
2	5·5	2·75000	0·1271704	334·46	326
3	3·33333	1·66667	0·2119511	557·43	548
4	2·25	1·12500	0·2384450	627·11	611
5	1·6	0·80000	0·1907560	501·69	519
6	1·16667	0·58333	0·7112737	292·65	307
7	0·85714	0·42857	0·0476886	125·42	133
8	0·625	0·31250	0·0149027	39·19	40
9	0·44444	0·22222	0·0033117	8·71	11
10	0·3	0·15000	0·0004968	1·31	2
11, 12	—	—	0·0000529*	0·14	0
Total	—	—	1·0000000	2,630·00	2,630

*Obtained from the identity : $f(11) + f(12) = 1 - \sum_{x=0}^{10} f(x)$.

Example 8·57. *Fit a Poisson distribution to the following data which gives the number of doddens in a sample of clover seeds.*

No. of doddens (x)	:	0	1	2	3	4	5	6	7	8
Observed frequency (f)	:	56	156	132	92	37	22	4	0	1

Solution. $\text{Mean} = \frac{1}{N} \sum fx = \frac{986}{500} = 1{\cdot}972$

Taking the mean of the given distribution as the mean of the Poisson distribution we want to fit, we get $\lambda = 1{\cdot}972$, and

$$p(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} ; x = 0, 1, 2, \dots, \infty \quad \Rightarrow \quad p(0) = e^{-\lambda} = e^{-1{\cdot}972}, \text{ so that}$$

$$\log_{10} p(0) = -1{\cdot}972 \log_{10} e = -1{\cdot}972 \times 0{\cdot}43429 = -0{\cdot}856419 = \bar{1}{\cdot}143581$$

$$\therefore \quad p(0) = \text{Antilog } (\bar{1}.1436) = 0{\cdot}1392$$

Using the recurrence formula (8·21), the various probabilities, *viz.*, $p(1), p(2), \dots,$ can be easily calculated as shown in the following table :

CALCULATIONS FOR EXPECTED POISSON FREQUENCIES

x	$\frac{\lambda}{x+1}$	$p(x)$	Expected frequency $N.p(x) = 500.\ p(x)$
0	1·972	0·13920	69·6000 ≃ 70
1	0·986	0·27455	137·2512 ≃ 137
2	0·657	0·27006	135·3296 ≃ 135
3	0·493	0·17793	88·9566 ≃ 89
4	0·394	0·10964	43·8556 ≃ 44
5	0·328	0·03459	17·2966 ≃ 17
6	0·281	0·01137	5·6846 ≃ 6
7	0·247	0·00320	1·6013 ≃ 2
8	0·219	0·00078	0·3942 ≃ 0

Remark. In rounding the figures to the nearest integer it has to be kept in mind that the total of the observed and the expected frequencies should be same.

Example 8.3 Fit a normal distribution to the frequency distribution of height of Indian adult males given in Table 3.10. Also draw the fitted curve over the histogram of the observed distribution.

For the distribution of height of Indian adult males, the mean and standard deviation were found to be

$$\bar{x} = 164 \cdot 734 \text{ cm. and } s = 5 \cdot 472 \text{ cm.}$$

Here $$n = 177 \text{ and } n/s = 32 \cdot 3462 \, .$$

TABLE 8.6

FITTING A NORMAL DISTRIBUTION TO THE HEIGHT-DISTRIBUTION OF INDIAN ADULT MALES (TABLE 3.10)

Height (cm.) x (1)	$\tau = (x - \bar{x})/s$ (2)	$\phi(\tau)$ (3)	Ordinate $= \frac{n}{s}\phi(\tau)$ (4)	$\Phi(\tau)$ (5)	$\Delta\Phi(\tau)$ (6)	Expected frequency $n \times \Delta\Phi(\tau)$ (7)	Observed frequency (8)
$-\infty$	$-\infty$	0	0	0			
					0·0001126*	0·020	0
144·55	−3·689	0·0004424	0·0143	0·0001126			
					0·0026478	0·469	1
149·55	−2·775	0·0084874	0·2745	0·0027604			
					0·0286123	5·064	3
154·55	−1·861	0·0706097	2·2839	0·0313727			
					0·1404492	24·860	24
159·55	−0·947	0·2547828	8·2412	0·1718219			
					0·3146168	55·687	58
164·55	−0·034	0·3987070	12·8966	0·4864387			
					0·3241316	57·371	60
169·55	0·880	0·2708640	8·7614	0·8105703			
					0·1530213	27·085	27
174·55	1·794	0·0798081	2·5815	0·9635916			
					0·0330236	5·845	2
179·55	2·708	0·0101984	0·3299	0·9966152			
					0·0032381	0·573	2
184·55	3·621	0·0005673	0·0183	0·9998533			
					0·0001467**	0·026	0
∞	∞	0	0	1			
Total	—	—	—	—	1·0000000	177·000	177

*It is the probability $P[x \leq 144{\cdot}55]$. **It is the probability $P[x \geq 184{\cdot}55]$.

With these, we can now compute the expected frequencies for the different class-intervals and the ordinates at the class-boundaries in the manner explained above. In the tables, $\phi(\tau)$ and $\Phi(\tau)$ are given for values of τ at intervals of 0·01 while in the present case we have taken $\tau = (x - \bar{x})/s$ correct to 3 decimal places. For obtaining $\phi(\tau)$ and $\Phi(\tau)$ for these values, we have used linear interpolation.

The agreement between the observed and the expected series of frequencies would seem to be fairly good. This agreement will also be apparent from Fig. 8.2, where we have the fitted normal curve, obtained on the basis of col. (4) of Table 8.6, superimposed on the histogram of the observed distribution.

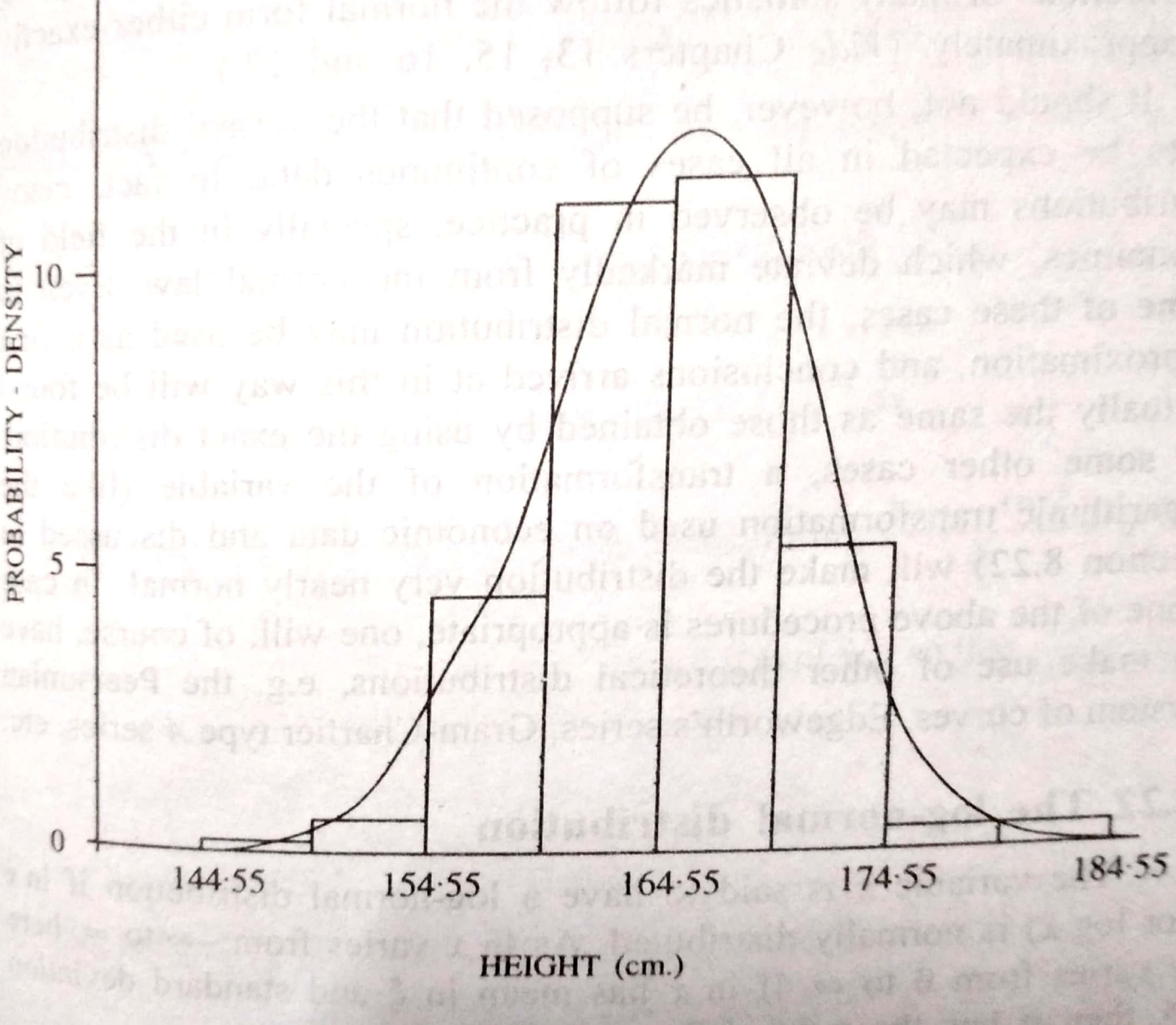

Fig. 8.2 Fitted normal curve together with the histogram of the height-distribution of Indian adult males (Table 3.10).

Example 8·63. *Given the hypothetical distribution :*

No. of cells (x)	:	0	1	2	3	4	5	Total
Frequency (f)	:	213	128	37	18	3	1	400

Fit a negative binomial distribution and calculate the expected frequencies.

Solution. Let X be negative binomial variate with parameters r and p.

$$\mu_1' = \text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{273}{400} = 0{\cdot}6825 = \frac{rq}{p} \quad ; (q = 1 - p) \qquad \dots (*)$$

$$\mu_2' = \frac{\Sigma fx^2}{\Sigma f} = \frac{511}{400} = 1{\cdot}2775$$

$$\mu_2 = \mu_2' - \mu_1'^2 = 1{\cdot}2775 - (0{\cdot}6825)^2 = 0{\cdot}8117$$

$$\therefore \quad \text{Variance} = 0{\cdot}8117 = \frac{rq}{p^2} \qquad \dots (**)$$

Dividing (*) by (**), we get $\quad p = \dfrac{0{\cdot}6825}{0{\cdot}8117} = 0{\cdot}8408, \; q = 1 - p = 0{\cdot}1592$

$$\therefore \quad r = \frac{p \times 0{\cdot}6825}{q} = \frac{0{\cdot}5738}{0{\cdot}1592} = 3{\cdot}6043 \simeq 4 \qquad \text{[From (*)]} \qquad \dots (***)$$

since, r being the number of successes cannot be fractional.

$$f_0 = p^r = ({\cdot}8408)^4 = 0{\cdot}4978 \simeq 0{\cdot}5$$

$$f_1 = \frac{r+0}{0+1} q f_0 = rq f_0 = 0{\cdot}5738 \times 0{\cdot}5 = 0{\cdot}2869$$

$$(\therefore rq = p \times 0{\cdot}6825 = 0{\cdot}5738 \qquad \text{[From (***)]}$$

$$f_2 = \frac{r+1}{1+1} \cdot q \cdot f_1 = \frac{5}{2} \times 0{\cdot}1592 \times 0{\cdot}2869 = 0{\cdot}1142$$

$$f_3 = \frac{r+2}{2+1} \cdot q \cdot f_2 = \frac{6}{3} \times 0{\cdot}1592 \times 0{\cdot}1142 = 0{\cdot}0364$$

$$f_4 = \frac{r+3}{3+1} \cdot q \cdot f_3 = \frac{7}{4} \times 0{\cdot}1592 \times 0{\cdot}0364 = 0{\cdot}0101$$

$$f_5 = \frac{r+4}{4+1} \cdot q \cdot f_4 = \frac{8}{5} \times 0{\cdot}1592 \times 0{\cdot}0101 = 0{\cdot}0026$$

$\therefore$ Expected frequencies are : $(N = 400)$

Nf_0	Nf_1	Nf_2	Nf_3	Nf_4	Nf_5
200	114·76	45·68	14·56	4·04	1·04

$\therefore$ *Observed Frequency*	:	213	128	37	18	3	1	1
Expected Frequency	:	200	115	46	14	4		1

etc.]

Theorem : 3. If $A_1, A_2, \ldots, A_n$ $(n \geq 2)$ are n events, then,

$$P\left(\sum_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{1\leq i<j\leq n} P(A_i A_j) + \sum_{1\leq i<j<k\leq n} P(A_i A_j A_k)$$

$$- \ldots\ldots + (-1)^{n-1} P(A_1 A_2 \ldots A_n) \tag{1.7}$$

Proof : We shall prove this theorem by the principle of induction.

Previously in Theorem 2, we have already seen that this theorem is true for $n = 2, 3$. Let us assume that this theorem is true for $n = m$. Now we shall establish that this theorem is also true for $n = m + 1$.

Now,

$$P\left(\sum_{i=1}^{m+1} A_i\right) = P\left(\sum_{i=1}^{m} A_i + A_{m+1}\right)$$

$$= P\left(\sum_{i=1}^{m} A_i\right) + P(A_{m+1}) - P\left\{\left(\sum_{i=1}^{m} A_i\right) A_{m+1}\right\} \quad \text{[By Theorem 2]}$$

$$= \sum_{i=1}^{m} P(A_i) - \sum_{1\leq i<j\leq m} P(A_i A_j) + \sum_{1\leq i<j<k\leq m} P(A_i A_j A_k)$$

$$- \ldots + (-1)^{m-1} P(A_1 A_2 \ldots A_m) + P(A_{m+1}) - P\left(\sum_{i=1}^{m} A_i A_{m+1}\right)$$

$$= \sum_{i=1}^{m+1} P(A_i) - \sum_{1\leq i<j\leq m} P(A_i A_j) + \ldots + (-1)^{m-1} P(A_1 A_2 \ldots A_m)$$

$$- \left[\sum_{i=1}^{m} P(A_i A_{m+1}) - \sum_{1\leq i<j\leq m} P(A_i A_j A_{m+1}) + \ldots + (-1)^{m-1} P(A_1 A_2 \ldots A_{m+1})\right]$$

$$= \sum_{i=1}^{m+1} P(A_i) - \left[\sum_{1\leq i<j\leq m} P(A_i A_j) + \sum_{i=1}^{m} P(A_i A_{m+1})\right]$$

$$+ \ldots + (-1)^{m} P(A_1 A_2 \ldots A_{m+1})$$

$$= \sum_{i=1}^{m+1} P(A_i) - \sum_{1\leq i<j\leq m+1} P(A_i A_j) + \ldots + (-1)^{m} P(A_1 A_2 \ldots A_{m+1})$$

Thus we see that if this theorem is true for $n = m$, then this is also true for $n = m + 1$.

But we have already seen that the formula is true for $n = 2, 3$; so this formula is true for $n = 3 + 1 = 4, 4 + 1 = 5$ etc.

So this theorem is true for any positive integral value of n.

Note : The result (1.7) can also be written as

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{1\le i<j\le n} P(A_i \cap A_j) +$$

$$\sum_{1\le i<j<k\le n} P(A_i \cap A_j \cap A_k) - \cdots\cdots + (-1)^{n-1} P(A_1 \cap A_2 \cap \cdots \cap A_n) \quad (1.7a)$$

Theorem : 4. If $\{A_n\}$ is a monotonic sequence of events, then $P\left(\lim_{n\to\infty} A_n\right) = \lim_{n\to\infty} P(A_n)$.

Proof : Case I. When the sequence $\{A_n\}$ is monotonic non-decreasing or expanding sequence of events.

In this case

$$A_1 \subseteq A_2 \subseteq A_3 \subseteq \cdots \subseteq A_n$$

and hence,

$$\sum_{i=1}^{n} A_i = A_n$$

i.e., $\lim A_n = \sum_{n=1}^{\infty} A_n$ .... (1)

It we consider another set of events $B_1, B_2, ..., B_n$, such that

$$B_1 = A_1, B_2 = A_2 - A_1, ..., B_n = A_n - A_{n-1} \ (n \ge 2),$$

then $\{B_n\}$ is a sequence of pairwise mutually exclusive events. In this case,

$$\sum_{n=1}^{\infty} A_n = \sum_{n=1}^{\infty} B_n \text{ and } \sum_{i=1}^{n} A_i = \sum_{j=1}^{n} B_j \quad .... (2)$$

Also, $P\left(\sum_{n=1}^{\infty} B_n\right) = \sum_{n=1}^{\infty} P(B_n)$ .... (3)

[Since $B_1, B_2, ...$ are pairwise mutually exclusive]

and for all positive integral values of n,

$$A_n = \sum_{i=1}^{n} B_i$$

Now, $P(\lim A_n) = P\left(\sum_{n=1}^{\infty} A_n\right)$ [by (1)]

$= P\left(\sum_{n=1}^{\infty} B_n\right)$ [by (2)]

$$= \sum_{1}^{\infty} P(B_n) \quad \text{[by (3)]}$$

$$= \lim \sum_{i=1}^{n} P(B_i) = \lim P\left(\sum_{i=1}^{n} B_i\right) = \lim P\left(\sum_{i=1}^{n} A_i\right)$$

$$= \lim P(A_n)$$

Thus, $P(\lim A_n) = \lim P(A_n)$.

Case II. When the sequence $\{A_n\}$ is monotonic non-increasing or contracting sequence of events.

In this case, $A_1 \supseteq A_2 \supseteq A_3 \supseteq \cdots \supseteq A_n$ and hence $\{\overline{A}_n\}$ is monotonic non-decreasing or expanding sequence. So, as derived in case I,

$$P\{\lim \overline{A}_n\} = \lim P(\overline{A}_n) \quad \ldots\ldots\ldots\ldots (4)$$

But, $P(\lim \overline{A}_n) = P(\overline{\lim A_n}) = 1 - P(\lim A_n)$

Hence from (4),

$$1 - P(\lim A_n) = 1 - \lim P(A_n)$$

or, $P(\lim A_n) = \lim P(A_n)$.

So, in either case,

$$P(\lim A_n) = \lim P(A_n)$$

1.15. Bernoulli's trials :

Let a random experiment be repeated n times. Then these repeated n trials are said to be Bernoulli's trials or n-Bernoulli trials if

(i) these trials are independent,

(ii) in each trial, there are only two possible outcomes, generally known as 'success' and 'failure',

(iii) probability of 'success' (or 'failure') remains the same for each trial.

An unbiased coin tossed n times is an example of a Bernoulli's trial. The event space consists of only two points 'head' (success) and 'tail' (failure) and in each trial, probability of getting head is $\frac{1}{2}$.